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$$p^0 : \frac{dx_0}{dt} - x_0 r_0 = 0 \quad \text{(A-16)}$$

$$p^1 : \frac{dx_1}{dt} - x_1 r_0 + r_1 U_0 x_0 + \frac{r_0 x_0^2}{k_0} - \frac{r_1 U_0 x_0^2}{k_0} + \frac{r_0 k_1 T_0 x_0^2}{k_0^2} - \frac{r_1 U_0 x_0^2 k_1 T_0}{k_0^2} + \frac{r_0^2 c x_0^2}{k_0} - \frac{r_0 r_1 c U_0 x_0^2}{k_0} + \frac{r_0^2 c k_1 T_0 x_0^2}{k_0^2} - \frac{r_0 r_1 c k_1 T_0 x_0^2 U_0}{k_0^2} \quad \text{(A-17)}$$
$$\frac{r_0^2 c x_0^3}{k_0^2} + \frac{r_0 c r_1 x_0^2 U_0}{k_0^2} - \frac{2 k_1 r_0^2 c x_0^3 T_0}{k_0^3} + \frac{2 r_0 c r_1 k_1 x_0^3 U_0 T_0}{k_0^3} + a_1 x_0 y_0 = 0$$

$$p^0 : \frac{dy_0}{dt} + d_1 y_0 = 0 \quad \text{(A-18)}$$

$$p^1 : \frac{dy_1}{dt} + d_1 y_0 - \beta_1 \alpha_1 x_0 y_0 + a_2 y_0 z_0 + \beta_{11} V_0 y_0 + b_1 y_0^2 = 0 \quad \text{(A-19)}$$

$$p^0 : \frac{dz_0}{dt} + d_2 z_0 = 0 \quad \text{(A-20)}$$

$$p^1 : \frac{dz_1}{dt} + d_2 z_0 - \beta_2 a_2 z_0 y_0 + \beta_{22} W_0 z_0 + c_3 z_0^2 = 0 \quad \text{(A-21)}$$

$$p^0 : \frac{dT_0}{dt} - \delta_0 T_0 = 0 \quad \text{(A-22)}$$

$$p^1 : \frac{dT_1}{dt} - \delta_0 T_1 - Q_0 + \alpha_1 x_0 T_0 = 0 \quad \text{(A-23)}$$

$$p^0 : \frac{dU_0}{dt} + \delta_1 U_0 = 0 \quad \text{(A-24)}$$

$$p^1 : \frac{dU_1}{dt} + \delta_1 U_1 - \alpha_1 x_0 T_0 + a_1 a_3 U_0 x_0 y_0 = 0 \quad \text{(A-25)}$$

$$p^0 : \frac{dV_0}{dt} + \delta_2 V_0 = 0 \quad \text{(A-26)}$$

$$p^1 : \frac{dV_1}{dt} + \delta_2 V_1 - a_1 a_3 U_0 x_0 y_0 + a_2 a_4 V_0 y_0 z_0 = 0 \quad \text{(A-27)}$$

$$p^0 : \frac{dW_0}{dt} + \delta_3 W_0 = 0 \quad \text{(A-28)}$$

$$p^1 : \frac{dW_1}{dt} + \delta_3 W_1 - a_2 a_4 V_0 y_0 z_0 = 0 \quad \text{(A-29)}$$

$$x_0 = L e^{r_0 t} \quad \text{(A-30)}$$

$$x_1(t) = [-\frac{r_1 LA}{\delta_1} + \frac{L^2}{k_0} - \frac{r_1 A L^2}{k_0(r_0-\delta_1)} + \frac{r_0 k_1 P L^2}{k_0^2(r_0-\delta_1)} - \frac{r_1 k_1 A P L^2}{k_0^2(r_0-\delta_1+\delta_0)} + \frac{r_0 c L^2}{k_0} - \frac{r_0 r_1 c A L^2}{k_0(r_0-\delta_1)} + \frac{r^2 c k_1 P L^2}{k_0^2(r_0+\delta_0)} - \frac{r_1 r_0 c A k_1 P L^2}{k_0^2(r_0-\delta_1+\delta_0)} - \frac{r_0 c L^3}{2k_0^2}$$
$$+ \frac{r_0 r_1 c A L^3 A}{k_0^2(2r_0-\delta_1)} - \frac{aLM}{d_1} - \frac{2k_1 r_0^2 c L^3 P}{k_0^3(2r_0+\delta_0)} + \frac{2 r_0 r_1 c k_1 A L^3 P}{k_0^3(2r_0-\delta_1+\delta_0)}] e^{r_0 t} + \frac{r_1 LA}{\delta_1} e^{(r_0-\delta_1)t} - \frac{L^2}{k_0} e^{2r_0 t}$$
$$+ \frac{r_1 A L^2}{k_0(r_0-\delta_1)} e^{(2r_0-\delta_1)t} - \frac{r_0 c L^2}{k_0} e^{2r_0 t} - \frac{r_0 k_1 P L^2}{k_0^2(r_0+\delta_0)} e^{(2r_0+\delta_0)t} + \frac{r_1 k_1 A P L^2}{k_0^2(r_0-\delta_1+\delta_0)} e^{(2r_0-\delta_1+\delta_0)t} + \frac{r_0 r_1 c A L^2}{k_0(r_0-\delta_1)} e^{(2r_0-\delta_1)t}$$
$$- \frac{r^2 c k_1 P L^2}{k_0^2(r_0+\delta_0)} e^{(2r_0+\delta_0)t} + \frac{aLM}{d_1} e^{(r_0-d_1)t} + \frac{r_0 c L^3}{2k_0^2} e^{3r_0 t} + \frac{r_1 r_0 c A k_1 P L^2}{k_0^2(r_0-\delta_1+\delta_0)} e^{(2r_0+\delta_0-\delta_1)t} - \frac{r_0 r_1 c A L^3 A}{k_0^2(2r_0-\delta_1)} e^{(3r_0-\delta_1)t} \quad \text{(A-31)}$$
$$+ \frac{2k_1 r_0^2 c L^3 P}{k_0^3(2r_0+\delta_0)} e^{(3r_0+\delta_0)t} - \frac{2 r_0 r_1 c k_1 A L^3 P}{k_0^3(2r_0-\delta_1+\delta_0)} e^{(3r_0-\delta_1+\delta_0)t}$$

$$y_0 = M e^{-d_1 t} \quad \text{(A-32)}$$

$$y_1(t) = -[\frac{\beta_1 \alpha_1 LM}{r_0} + \frac{a_2 MN}{d_2} + \frac{\beta_{11} BM}{\delta_2} + \frac{b_1 M}{d_1}] e^{-d_1 t} + \frac{\beta_1 \alpha_1 LM}{r_0} e^{(r_0-d_1)t} + \frac{a_2 MN}{d_2} e^{-(d_1+d_2)t} + \frac{\beta_{11} BM}{\delta_2} e^{-(\delta_2+d_1)t} + \frac{b_1 M^2}{d_1} e^{-2d_1 t} \quad \text{(A-33)}$$

$$z_0 = N e^{-d_2 t} \quad \text{(A-34)}$$

$$z_1(t) = [\frac{\beta_2 a_2 MN}{d_1} - \frac{\beta_{22} DN}{\delta_3} - \frac{c_3 N^2}{d_2}] e^{-d_2 t} - \frac{\beta_2 a_2 MN}{d_1} e^{-(d_1+d_2)t} + \frac{c_3 N^2}{d_2} e^{-2d_2 t} + \frac{\beta_{22} DN}{\delta_3} e^{-(\delta_3+d_2)t} \quad \text{(A-35)}$$

$$T_0 = P e^{\delta_0 t} \quad \text{(A-36)}$$

$$T_1(t) = [\frac{Q_0}{\delta_0} + \frac{\alpha_1 LP}{r_0}] e^{\delta_0 t} - \frac{Q_0}{\delta_0} - \frac{\alpha_1 LP}{r_0} e^{(r_0+\delta_0)t} \quad \text{(A-37)}$$

$$U_0 = A e^{-\delta_1 t} \quad \text{(A-38)}$$

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$$U_1(t)=[\frac{-\alpha_1 LP}{r_0+\delta_0+\delta_1}+\frac{a_1 a_3 ALM}{r_0-d_1}]e^{-\delta_1 t} + \frac{\alpha_1 LP}{r_0+\delta_0+\delta_1}e^{(r_0+\delta_0)t} - \frac{a_1 a_3 ALM}{r_0-d_1}e^{(-\delta_1+r_0-d_1)t} \quad \text{(A-39)}$$

$$V_0 = Be^{-\delta_2 t} \quad \text{(A-40)}$$

$$V_1(t)=[-\frac{a_1 a_3 ALM}{r_0-d_1-\delta_1+\delta_2}-\frac{a_2 a_4 BMN}{d_1+d_2}]e^{-\delta_2 t} + \frac{a_1 a_3 ALM}{r_0-d_1-\delta_1+\delta_2}e^{(-\delta_1+r_0-d_1)t} + \frac{a_2 a_4 BMN}{d_1+d_2}e^{-(d_1+d_2+\delta_2)t} \quad \text{(A-41)}$$

$$W_0 = De^{-\delta_3 t} \quad \text{(A-42)}$$

$$W_1(t) = [\frac{a_2 a_4 BMN}{d_1+d_2+\delta_2-\delta_3}]e^{-\delta_3 t} + \frac{a_2 a_4 BMN}{\delta_3-d_1-d_2-\delta_2}e^{-(\delta_{2+}d_1+d_2)t} \quad \text{(A-43)}$$

$$x(t) = \lim_{p\to 1} x(t) = x_0 + x_1 + ... \quad \text{(A-44)}$$

$$y(t) = \lim_{p\to 1} y(t) = y_0 + y_1 + ... \quad \text{(A-45)}$$

$$z(t) = \lim_{p\to 1} z(t) = z_0 + z_1 + ... \quad \text{(A-46)}$$

$$T(t) = \lim_{p\to 1} T(t) = T_0 + T_1 + ... \quad \text{(A-47)}$$

$$U(t) = \lim_{p\to 1} U(t) = U_0 + U_1 + ... \quad \text{(A-48)}$$

$$V(t) = \lim_{p\to 1} V(t) = V_0 + V_1 + ... \quad \text{(A-49)}$$

$$W(t) = \lim_{p\to 1} W(t) = W_0 + W_1 + ... \quad \text{(A-50)}$$

After putting Eqns. (A-30) and (A-31) into Eqns. (A-44), (A-32) and (A-33) into Eqns. (A-45), (A-34) and (A-35) into Eqns. (A-46), (A-36) and (A-37) into Eqns. (A-47), (A-38) and (A-39) into Eqns. (A-48), (A-40) and (A-41) into Eqns. (A-49), (A-42) and (A-43) into Eqns. (A-50). We can obtain the final results which can be described in Eqns. (9-15).

APPENDIX B

MATLAB Program to find the numerical solution of the Eqns. (1)- (8)

```
function viji2
options= odeset('RelTol',1e-6,'Stats','on');
%initial conditions
Xo = [0.0001;0.0001;0.0001;0.0001;0.0001;0.0001;0.0001];
tspan = [0,20];
tic
[t,X] = ode45(@TestFunction,tspan,Xo,options);
toc
figure
hold on
plot(t, X(:,1), 'red')
plot(t, X(:,2), 'black')
plot(t, X(:,3), 'green')
plot(t, X(:,4), 'rose')
plot(t, X(:,5), 'yellow')
plot(t, X(:,6), 'black')
plot(t, X(:,7), 'm')
legend('x1','x2','x3','x4','x5','x6','x7')
ylabel('x - Population')
xlabel('t - Time')
return
function [dx_dt]= TestFunction(~,x)
r0=5.66; r1=11.0; k0=16.2; k1=3.0; c=4.25; c3=1.02;
alpha1=2.12; b1=1.231; beta1=1.2; beta11=1.1; beta2=1.6;
beta22=1.15; a1=3.22; a2=2.13; a3=2.865; a4=4.21;
delta0=7.52; delta1=2.5; delta2=3.99; delta3=1.3; Q0=2.988;
d1=1.45; d2=1.49;
dx_dt(1)=(x(1)*(r0-(r1*x(5))))*(((k0-(k1*x(4)))-x(1))/((k0-
(k1*x(4)))+(r0*c*x(1))))-(a1*x(1)*x(2));
dx_dt(2)=(beta1*a1*x(1)*x(2))-(a2*x(2)*x(3))-(beta11* x(6)*
x(2)) -(d1*x(2))-(b1*(x(2))^2);
dx_dt(3)=(beta2*a2*x(2)*x(3))-(beta22*x(7)*x(3))-(d2*x(3))-
(c3*(x(3))^2);
dx_dt(4) = Q0-( delta0*x(4))-(alpha1*x(1)*x(4));
dx_dt(5)=(alpha1*x(1)*x(4))-(delta1*x(5))-
(a3*x(5)*a1*x(1)*x(2));
dx_dt(6) =(a3*x(5)*a1*x(1)*x(2))-(delta2*x(6))-
(a4*x(6)*a2*x(2)*x(3));
dx_dt(7) =(a4*x(6)*a2*x(2)*x(3))-(delta3*x(7));
dx_dt = dx_dt';
return
```

APPENDIX C

NOMENCLATURE

SYMBOL	**MEANING**
x	prey population of density
y	intermediate predator population of density
z	top predator population of density
T	toxicant concentration in the environment
U	toxicant concentration in the prey population
V	toxicant concentration in the intermediate predator population
W	Toxicant concentration in the top predator population
$K(T)$	carrying capacity of prey
d_1	death rates of intermediate predator
d_2	death rates of top predator
$r(U)$	specific growth rate of prey population

β_1 and β_2	conversion coefficients
$\beta_3(U)$ and $\beta_4(U)$	toxicant transfer function
Q_0	toxicant rate of introduction into the environment
$\delta_0, \delta_1, \delta_2, \delta_3$	rate of toxicants in the environment as well as in the populations
β_{11} and β_{22}	death rates of predators due to organismal toxicant concentration
k_0	natural carrying capacity
k_1	rate of decrease carrying capacity
r_0	intrinsic growth rate of prey
r_1	growth rate of prey population
α_1	depletion rate of toxicant in the environment
c, a_1, a_2, a_3, a_4	positive constants
$\beta_3(U)$	a_3U
$\beta_4(V)$	a_4V
$r(U)$	$r_0 - r_1U$
$K(T)$	$k_0 - k_1T$

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