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constant, $a(z) = (z - z_0)^{\beta}, \beta > 0$ is constant. It is easy to see that function

$$u(x,z) = u_0 + \sum_{i=1}^{m} c_i \int_0^{x_i} a_i(t)dt + c\int_0^{z} a(t)$$

is a solution of the equation (10). Here c_i, c are the solutions of equations (****) under some m, n; u_0 is defined from (8). This solution equals to the solution of next type:

$$u(x,z) = u_0 + \sum_{i=1}^{m} \int_0^{x_i^{(n+1)/n}} a_i(t)dt + \int_0^{z^{(n+1)/n}} a(t)dt,$$

which satisfies the equation (10) with coefficients $a_i(x_i^{(n+1)/n}$ and $a(z^{(n+1)/n})$, $(x,z) \in G$, $m \geq 2$, $n = 1, 2....$

The equation of th k-th order. We consider next equation[15]:

$$\sum_{i=1}^{m} \left(\frac{\partial^k u}{\partial x_i^k}\right)^n = \left(\frac{\partial^k u}{\partial z^k}\right)^n,$$

General solution of this equation is represented in the form of:

$$u(x_1, ..., x_m, z) = \varphi(x_1, ..., x_m, z) + \sum_{i=1}^{m} c_i \frac{x_i^k}{k!} + \frac{cz^k}{k!},$$

or $u(x_1, ..., x_m, z) = \varphi(x_1, ..., x_m, z) + \sum_{i=1}^{m} \frac{x_i^{k+2/n}}{(1+2/n)...(k+2/n)} + \frac{z^{k+2/n}}{(1+2/n)...(k+2/n)}$, where $\sum_{i=1}^{m} x_i^2 = z^2$, $-\infty < x_i < \infty$, $-\infty < z < \infty$, c_i, c are the solutions of the equation (1), the function $\varphi(.)$ is polinom of the $k-1$-th order. Coefficients of this polinom we shall define with help of boundary and initial conditions. For example at $m = 2, k = 2$ if we are given

$$\frac{\partial u}{\partial z}\mid_{z=0} = u_1(x_1, x_2), u\mid_{z=0} = u_0(x_1, x_2)$$

for definition of the function $\varphi(.)$ we have: $\varphi(x_1, x_2, z) = u_0(0,0) + u_1(0,0)z + \frac{\partial u_0(0,0)}{\partial x_1}x_1 + \frac{\partial u_0(0,0)}{\partial x_2}x_2 + \frac{\partial u_1(0,0)}{\partial x_1}x_1 z + + \frac{\partial u_1(0,0)}{\partial x_2}x_2 z + \frac{\partial^2 u_0(0,0)}{\partial x_1 \partial x_2}x_1 x_2 + \frac{\partial^2 u_1(0,0)}{\partial x_1 \partial x_2}x_1 x_2 z.$

Corollary 2.. *The solutions of considering equations with constant coefficients and connected with predetermined system (***) are polinoms of k-th order, where k equals to the order of these differential equations. The leading coefficients satisfy the equations of (1) and the others are defined from corresponding boundary and initial conditions.*

Remark 1. All solutions of the equations (1): $Z^n = \sum_{i=1}^{m} X_i^n$ are represented in the following way: $X_i = k^{1/n}\alpha_i^{1/(n-s)}, i = 1, ..., m-1$ and $X_m = k^{1/n}(1 - \sum_{i=1}^{m-1} \alpha_i^{\frac{n}{n-s}})^{1/n}, Z = k^{1/n}$. Really, we consider the function $\mu_s(\alpha) = [\sum_{i=1}^{m} \alpha_i X_i^s]^{1/s}, \alpha \in A$, $A = [\alpha_i : 0 < \alpha_i < 1, \sum_{i=1}^{m} \alpha_i^{\frac{n}{n-s}} = 1]$, and solving the problem of $\max_{\alpha \in A} \mu_s(\alpha)$ we have mentioned above solutions of the equations (1). It is clear that these solutions are not positive integer numbers under $n > 2$.

In work the new model of growth of the population, so-called energetic model of growth for population $\frac{dL}{dt} = \delta\, L,\ \delta : \int_G B(a)e^{-\lambda\, a}da = 1, \quad \lambda = \delta + \beta + \gamma\, r, \quad L(t) = \left(\sum_{j=0}^{\infty} c_j^p e^{\alpha_j\, t}\cos\left(\beta_j t\right)\right)^{1/p}$,(p=2),

is constructed and energetic theory of population growth is also investigated. This model is received for one and n countries, where $L(t) = \left(\int_G \varphi(\eta)\, N^p(\eta)\, d\eta\right)^{1/p}$, $0 < p < \infty$. Here

$B(a) = B_0(a)e^{-p\int_0^a F_0(\xi)d\xi}$ is function of survival rate of the population, $\varphi\,(\eta)$ is some non-negative function with a condition, $\varphi(x,a,t) = \int_a^{\infty} e^{-\int_a^{\xi} F_o^*(\eta)d\eta + \delta(a-\xi)} B_0^*(\varsigma)\,\varphi(x + r(\varsigma - a), 0, t - a + \xi)d\xi$, $\int_G \varphi\,(\eta)\,d\eta = 1$; $N(\eta) = N\,(x, a,\, t)$, $N(\eta) = \sqrt[p]{\frac{\varphi\,(\eta)}{\int_0^{\infty}\varphi^2\,(\varsigma)\,d\varsigma}}\ L\,(t)$, and values of α_j β_j - are solutions of system $\int_G B(a)e^{-\alpha_j a}\cos\beta_j a\, da = 1$, $\int_G B(a)e^{-\alpha_j a}\sin\beta_j a\, da = 0.$

It is should out that energetic model is constructed on the base of some initial groups of the models, describing growth of a population in view of age structure and spatial distributions:

1.

$$\begin{cases} \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) N = -F_0(a)N,\ 0 < a < \infty,\ 0 < t < t_r \\ N\,(a,\,0) = N_0(a), \quad 0 \leq a < \infty, \\ N(0,\,t) = \sqrt[p]{\int_0^{\infty} B_0(a)N^p(a,t)\,da}, \\ \quad (a,t) \in G \end{cases}$$

2.

$$\begin{cases} \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a} + r\frac{\partial}{\partial x}\right) N = -F_0(a)N,\ 0 < a < \infty, 0 < t < t_k \\ N\,(x, a,\,0) = N_0(x,a), \quad 0 \leq a < \infty,\ 0 < x < L, \\ N(x, 0,\,t) = \sqrt[p]{\int_0^{\infty} B_0(a)N^p(x,a,t)\,da},\ 0 < x < L, \\ N\mid_{x=0} = 0 = N\mid_{x=0}\ , \end{cases}$$

3. *Models in view of a diffusion distributions* $,(x,a,t) \in G$.

The series of computer experiments were carried out for initial functions described
with help of uniformly and normal distributed laws.

Remark 2. The method is also used for differential equations of type:

$$\sum_{i=1}^{m} \left(\frac{\partial^k u}{a_i(x_i)\partial x_i^k}\right)^n = \left(\frac{\partial^p u}{a(z)\partial z^p}\right)^n,$$

$$\sum_{i=1}^{m} \left(\frac{\partial^k u}{a_i(x_i)\partial x_i^{k_1}\partial x_i^{k_2}}\right)^n = \left(\frac{\partial^p u}{\partial z^p}\right)^n,$$

$$\sum_{i=1}^{m} \left(\frac{\partial^i u}{a_i(x_i)\partial x_i^i}\right)^n = \left(\frac{\partial^p u}{a(z)\partial z^p}\right)^n, ...,$$

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where $k = 1, 2, 3..., p = 1, 2, 3..., k_1 + k_2 = k$ and others may be also solved with help of this method.

Example 2. Let $Lu = i\frac{\partial u}{\partial t}$, $L_j u = \sum_{j=1}^{m} \frac{\partial^k u}{\partial x_j^k}$ then we have $u = v + iw$, where

$v = u_0(x_1, x_2, t)\cos(\int_0^t F(a)da) + +c_0 \int_0^t a^{2/n}\cos(\int_a^t F(a')da')da,$

$w = -u_0(x_1, x_2, t)\sin(\int_0^t F(a)da) - c_0 \int_0^t a^{2/n}\sin(\int_a^t F(a')da')da + \frac{\sum_{j=1}^{m} x_j^{k+2/n}}{(1+2/n)...(k+2/n)}$ and $u_0(.)$ is polinom of $(k-1)$-th order, $F(t)$ is given function, $(x_1, ..., x_m, t) \in G$,

$G = \{\sum_{j=1}^{m} x_j^2 = c_0^n t^2\}$

4. Computer experiments of the equation $\sum_{j=1}^{m} X_j^n = Z^n$

Consider the case $n = 2$ and define positive integer solutions of the equations (1) under different values $m = 3, 4, 5, 6....$ It is clear that basic solution is the solution $x^2 + y^2 = z^2$. For example $x = 3, y = 4, z = 5$.

Computer experiments are given in the form of:

$m=3$	$m=4$	$m=5$
$X_1 = 9$	$X_1 = 27$	$X_1 = 81$
$X_2 = 12$	$X_2 = 36$	$X_2 = 108$
$X_3 = 20$	$X_3 = 60$	$X_3 = 180$
$Z = 25$	$X_4 = 100$	$X_4 = 300$
	$Z = 125$	$X_5 = 500$
		$Z = 625$

$m=9$	$m=10$	$m=11$
$X_1 = 6561$	$X_1 = 19683$	$X_1 = 59049$
$X_2 = 8748$	$X_2 = 26244$	$X_2 = 78732$
$X_3 = 14580$	$X_3 = 43740$	$X_3 = 131220$
$X_4 = 24300$	$X_4 = 72900$	$X_4 = 218700$
$X_5 = 40500$	$X_5 = 121500$	$X_5 = 364500$
$X_6 = 67500$	$X_6 = 202500$	$X_6 = 607500$
$X_7 = 112500$	$X_7 = 337500$	$X_7 = 1012500$
$X_8 = 187500$	$X_8 = 562500$	$X_8 = 1687500$
$X_9 = 312500$	$X_9 = 937500$	$X_9 = 2812500$
$Z = 390625$	$X_{10} = 1562500$	$X_{10} = 4687500$
	$Z = 1953125$	$X_{11} = 7812500$
		$Z = 9765625$
...	....	....
...	....	....

Computer experiments under $n = 3, 4, ...,$ $s = 1.$**1.** $n = 3$. Under all experiments in this case we shall consider that the basic solution is equals to $x = 3, z = 9$. Then we have:

Under $m = 2; x_i : 3; 8.887488; z = 9; \sum x_i^3 = 729.0001; z^3 = 729$

Under $m = 5$; x_i : 81; 239.9622; 719.8866; 2159.66; 6478.979; $z = 6561$

$\sum x_i^3 = 2.824298.10^{11}, z^3 = 2.824298.10^{11}$

Under $m = 9$; x_i : 6561; 19436.94; 58310.81; 174932. 4; 524797.3; 1574392 4723176; $1,416953.10^7; 4,250858.10^7; z = 4,304672.10^7$

$\sum x_i^3 = 7,976644.10^{22}; z^3 = 7,976644.10^{22}$.

Under $m = 2; z = 100; \alpha = 0.1;$ $x_1 = 31.62278; x_2 = 98.93459; z = 100$

$\sum x_i^3 = 999999.9; z^3 = 1000000$

2. $n = 5$ The basic solution: $x = 3, z = 9$

Under $m = 2$; x_i: 3, 8,9928; $z = 9$;

$\sum x_i^5 = 59049$; $z^5 = 59049$

Under $m = 3$; x_i: 9; 26.97774; 80.93323; $z = 81$

$\sum x_i^5 = 3,486785.10^9; z^5 = 3,486785.10^9$

Under $m = 5$; x_i: 81; 242,7997; 728,399; 2185,197; 6555,591; $z = 6561$

$\sum x_i^5 = 1,215767.10^{19}; z^5 = 1,215767.10^{19}$

Under $m = 2; z = 100; \alpha = 0.1;$ $x_1 = 56.23413; x_2 = 98.84913; z = 100$

$\sum x_i^5 = 9,999999.10^9; z^5 = 10^{10}$

3. $n = 10$ The basic solution: $x = 5, z = 8$

Under $m = 2$; x_i : 5, 7,992694; $z = 8$;

$\sum x_i^{10} = 1,073742.10^9;$ $z^{10} = 1,073742.10^9$

Under $m = 3$; x_i : 25; 39,96347; 63,94155; $z = 64$

$\sum x_i^{10} = 1,152921.10^{18}; z^{10} = 1,152922.10^{18}$

Under $m = 5$; x_i : 625; 999,0868; 1598,539; 2557,662; 4092,259; $z = 4096$

$\sum x_i^{10} = 1,329228.10^{36}; z^{10} = 1,329228.10^{36}$

Under $m = 2; z = 100; \alpha = 0.1;$ $x_1 = 77.42637; x_2 = 99.139736$

$\sum x_i^{10} = 9,999999.10^{19}; z^{10} = 10^{20}$

Under $n = 15; m = 2; z = 100; \alpha = 0.1;$ $x_1 = 84.83429; x_2 = 99.41074$

$\sum x_i^{15} = 10^{30}; z^{15} = 10^{30}$

The dependence of the solutions from n . Now we shall consider the case when parameters m, α, $z = k^{1/n}$ are constants and we define the dependence of the solutions (1) from n.

1. Let $m = 2$, $\alpha = .5$, $k^{1/n} = 1$, then we have:

$n = 3$: $x_1 = .7071067$, $x_2 = .884657$;

$n = 100$: $x_1 = .993027$, $x_2 = .9931616$;

$n = 1000$: $x_1 = .993064$, $x_2 = .993078$;

$n = 2000$: $x_1 = .9996533$, $x_2 = .9996536$.

2. Let $m = 2$, $\alpha = .1$, $k^{1/n} = 1$, then we have:

$n = 3$: $x_1 = .3162278$, $x_2 = .989346$;

$n = 1000$: $x_1 = .9976978$, $x_2 = .9998949$;

$n = 10000$: $x_1 = .9997697$, $x_2 = .9999894$;

$n = 100000$: $x_1 = .999977$, $x_2 = .9999989$;

$n = 1000000$: $x_1 = .9999977$, $x_2 = .9999999$;

$n = 10000000$: $x_1 = .9999998$, $x_2 = 1$.

3. $m = 2$, $\alpha = .5$, $k^{1/n} = 3$, then:

$n = 3$: $x_1 = 2.12132$, 2.593973;

$n = 20$: $x_1 = 2.892528$, $x_2 = 2.902913$;

$n = 40$: $x_1 = 2.947152$, $x_2 = 2.949749$;

$n = 60$: $x_1 = 2.964962$, $x_2 = 2.966116$;

$n = 80$: $x_1 = 2.973793$, $x_2 = 2.974443$.

From these experiments it follows that under increase value of n the solutions of the equation (1) tend to value of z. When $\alpha = .5$ all x_i tend to z simultaneously and when $\alpha < .5$ then x_2 rapidly tend to z than x_1.

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