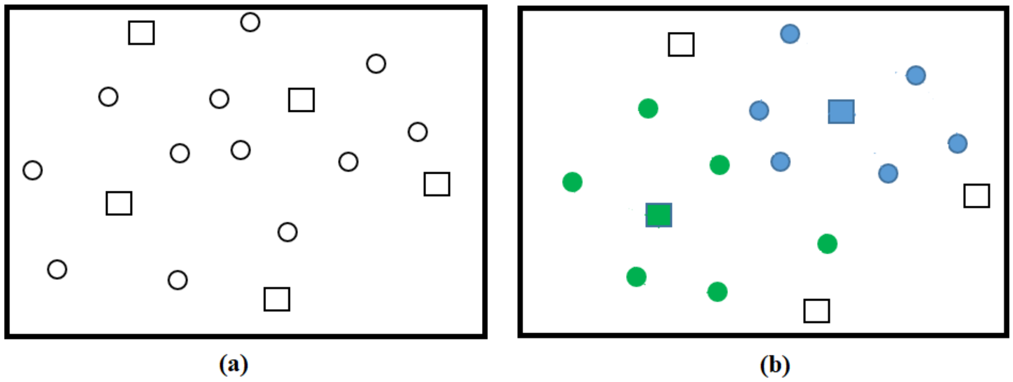

Fig. 1. An illustrative example of the problem [14].

TABLE I
Used notations.

Objective functions:	
f	Objective function defined based on the equilibrium of distances in sectors
Sets and indexes:	
$i \in \{1, ..., \bar{I}\}$	Index of customers
$j \in \{1, ..., \bar{J}\}$	Index of DCs and sectors
Parameters:	
D_{ij}	Euclidean distance between customer i and DC j
$\bar{D}$	Average distance of customers from DCs
r_{ij}	Binary parameter about if customer i is in the covering radius of DC j or not
De_i	Demand of customer i
$\bar{De}$	Average demand of customers in sectors
τ_{equ}	Tolerance for the equilibrium criteria
τ_r	Tolerance for the covering radius of DCs
n_{max}	Upper limit for the number of opened DCs
Variables:	
D_j	Total distance of customers from DC j in sector j
De_j	Total demand of customers in sector j
Binary decision variables:	
Y_j	Decision variable about if DC j is opened or not
X_{ij}	Decision variable about if customer i belongs to sector j or not
Positive decision variables:	
A_j	Positive variable to linearize the objective function
B_j	Positive variable to linearize the objective function

$$Y_j = \begin{cases} 1, & \text{if } DC\ j\ is\ opened \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

$$X_{ij} = \begin{cases} 1, & \text{if } customer\ i\ is\ in\ sector\ j \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

There is a DC in each sector, so if customer i is in sector j, which likewise implies that it is assigned to DC j.

The objective function is defined as in Equation 3, which is related to equilibrium of distances in sectors:

$$f = Min \sum_{j=1}^{\bar{J}} |D_j - \bar{D}| \quad (3)$$

where $D_j = \sum_{i=1}^{\bar{I}} D_{ij} \times X_{ij} \quad \forall \in \bar{J}$ and $\bar{D} = \sum_{i=1}^{\bar{I}} \sum_{j=1}^{\bar{J}} \frac{D_{ij}}{\bar{J}}$. It should be noted that $\bar{D}$ is not dependent on decision variables. Hence, it is calculated as a parameter, not a variable.

As defined in Constraint 4, at least one customer is assigned to each sector.

$$\sum_{i=1}^{\bar{I}} X_{ij} \geq 1, \quad \forall \in \bar{J} \quad (4)$$

In the reference [13], two definitions of compactness are given, one of them as an objective function and the other one as a constraint. Since these definitions are based on the total distance of the customers from the assigned DC, they cannot measure the accessibility of each customer to the DC. In a different way, we use constraint 4, which is a linear Constraint, to ensure the accessibility of each customer from the related DC. The linearity of the constraint also makes it easy to manage.

ISSN: 2692-5079

Each customer is assigned to only one sector, which is supplied by Constraint 5:

$$\sum_{j=1}^{\bar{J}} X_{ij} = 1, \ \forall i \in \bar{I} \tag{5}$$

Constraint 6 is to ensure that all customers are on the covering radius of at least one of the opened DCs.

$$\sum_{j=1}^{\bar{I}} r_{ij} Y_j \geq 1, \ \forall i \in \bar{I} \tag{6}$$

r_{ij} is a binary parameter, which is equal to one if customer i is in the covering radius of DC j, and otherwise is equal to zero. r_{ij} is calculated based on τ_r. Details are given in the section of experimental results.

Customers only receive service from one of the opened DCs. However, as defined in Constraint 6, each customer is in the covering radius of more than one DC. For the customers, this can be an advantage in emergency situations, which can be a case in which the DC that the customers in the assigned sector ordinarily receive service from, is damaged.

Customer i receives service from the opened DC j only if it is in its covering radius, which is defined as in Constraint 7.

$$X_{ij} \leq r_{ij}, \ \forall i \in \bar{I} \ and \, \forall j \in \bar{J} \tag{7}$$

Customer i can be assigned to DC j and receives service from it only if DC j is open, which is guaranteed by Constraint 8.

$$X_{ij} \leq Y_j, \ \forall i \in \bar{I} \ and \ \forall j \in \bar{J} \tag{8}$$

DC j is open if at least one customer is assigned to it, which is provided by Constraint 9.

$$Y_j \leq \sum_{i=1}^{\bar{I}} X_{ij}, \ \forall j \in \bar{J} \tag{9}$$

Applying Constraint 10, an upper limit can be specified for the number of opened DCs.

$$\sum_{i=1}^{\bar{J}} Y_j \leq n_{max} \leq \bar{J} \tag{10}$$

It is expected that the equilibrium be provided in terms of demands between the formed sectors, which is afforded by Constraint 11.

$$|De_j - \bar{De}| \leq \bar{De}(1 - \tau_{equ}), \quad \forall j = 1, ..., \bar{J}, \quad 0 \leq \tau_{equ} \leq 1 \tag{11}$$

where $De_j = \sum_{i=1}^{\bar{I}} De_i \times X_{ij}$, $\bar{De} = \sum_{i=1}^{\bar{I}} \sum_{j=1}^{\bar{J}} \frac{De_j}{\bar{J}}$.

III. Implementation

In this section, we present some details about implementation, which is done in the Pulp library of Python on an Intel Core i7 processor, 1.8 GHz with 16 GB of RAM.

In Python, after installing Pulp library, it can be imported using the following command:

$from \ pulp \ import \ *$

Since the Pulp library solves only linear models, using positive variables A_j and B_j the objective function, which is defined as in Equation 3, is linearized.

$$D_j - \bar{D} - A_j + B_j = 0, \ \forall j = 1, ..., \bar{J} \tag{12}$$

$$A_j \ and \ B_j \geq 0, \ \forall j = 1, ..., \bar{J} \tag{13}$$

$$Min \ f = \sum_{j=1}^{\bar{J}} A_j + \sum_{j=1}^{\bar{J}} B_j \tag{14}$$

More details about linearization can be found at the reference [14].

Sets of customers and DCs are defined as follows:
$setI = range(1, \bar{I})$
$setJ = range(1, \bar{J})$

Decision varaiable X_{ij} is defined as follows:
$xVars = LpVariable.dicts(name = "xVars", indexs = (setI, setJ), lowBound = 0, upBound = 1, cat = LpInteger)$

In the reference [14], although in the problem definition part more than one binary decision variable is introduced, only one of them is used in the solution process, which is X_{ij}. In this study, we use Y_j too, which is defined as in Equation 1. It is included in the model as follows:

$yvars = LpVariable.dicts(name = "yvars", indexs = setJ, lowBound = 0, upBound = 1, cat = LpInteger)$

Also, the positive variables used for linearization are defined as follows:
$A = LpVariable.dicts(name = "A", indexs = setJ, lowBound = 0, cat = LpInteger)$

$B = LpVariable.dicts(name = "B", indexs = setJ, lowBound = 0, cat = LpInteger)$

The name and type of the implemented model are defined as follows:
$prob = LpProblem("MIP\_Model", LpMinimize)$

Constraints are included to the model as follows:
Constraint 4:
$for\ j\ in\ setJ:$
$prob += lpSum(xVars[i][j]\ for\ i\ in\ setI)\ >=\ 1,""$

Constraint 5:
$for\ i\ in\ setI:$
$prob += lpSum(xVars[i][j]\ for\ j\ in\ setJ) == 1,""$

Constraint 6:
$for\ j\ in\ setJ:$
$prob += lpSum(r[i,j] * Y[j]\ for\ i\ in\ setI)\ >=\ 1,""$

Constraint 7:
$for\ i\ in\ setI:$
$for\ j\ in\ setJ:$
$prob += xVars[i][j]\ <=\ r[i,j],""$

Constraint 8:
$for\ i\ in\ setI:$
$for\ j\ in\ setJ:$
$prob += xVars[i][j]\ <=\ yVars[j],""$

Constraint 9:
$for\ j\ in\ setJ:$
$yVars[j]\ <=\ lpSum(xVars[i][j]\ for\ i\ in\ setI),""$

Constraint 10:
$lpSum(yVars[j]\ for\ j\ in\ setJ)\ <=\ nMax,""$

As the constraints defined based on the absolute value cannot be employed directly in Pulp, Constraint 11 is managed with two following constraints:

$for\ j\ in\ setJ:$
$prob += lpSum(de[i] * xVars[i][j]\ for\ i\ in\ setI) - deBar\ <=\ dej[j] * (1 - tau),""$

$for\ j\ in\ setJ:$
$prob += lpSum(-de[i] * xVars[i][j]\ for\ i\ in\ setI) + deBar\ <=\ dej[j] * (1 - tau),""$

where $deBar$ and $dej[j]$ are $\bar{De}$ and De_j in TABLE I, respectively.
Constraint 12:
$for\ j\ in\ setJ:$
$prob += lpSum(d[i,j] * xVars[i][j]\ for\ i\ in\ setI) - dBar - A[j] + B[j] == 0,""$

where $dBar$ is $\bar{D}$ in TABLE I.

The linearized objective function illustrated as in Equation 14, is appended to the model as follows:
$prob += lpSum(A[j]\ for\ j\ in\ setJ) + lpSum(B[j]\ for\ j\ in\ setJ)$

But the real value of the model's objective function, determined in Equation 3, is acquired like following:
$sum = 0$
$for\ j\ in\ setJ:$
$for\ i\ in\ setI:$
$sum += abs(d[i,j] * xVars[i][j].varValue - dBar)$

We utilize the solver $GLPK\_CMD$ in the Pulp library using the following command:
$prob.solve(GLPK\_CMD())$

IV. Experimental Results

Three benchmarks are created as 30×5, 450×75 and 900×150, which are indicated as the *Number of customers × Number of potential DCs*. To calculate the value of r_{ij}, we use parameter τ_r. Suppose that the maximum distance between all customers and DCs in a benchmark is d_{max}. In this case, if the distance between customer i and DC j is equal to or less than $d_{max} \times \tau_r$, r_{ij} is equal to one, otherwise, it is equal to zero. The value of one for r_{ij} means that customer i is reachable from DC j, thus is assignable to it.

In the benchmarks, two-dimensional coordinates of customers and DCs and also customers' demands, are generated according to $N(50;10)$ and $U(100;10)$, which are normal and discrete uniform distributions, respectively.

In Table II, the results are given according to the different values of τ_{equ} and benchmark size. Parameter τ_{equ} varies within the interval [0, 1]. At first, it is checked in which range of this parameter feasible solutions can be obtained, and then the tightest value is selected for each benchmark. The outcomes are given in TABLE II, which are for the case in which $n_{max} = \bar{J}$. The results in the table are for $\tau_r = 0.5$, but the same results are obtained for $\tau_r = 0.33$.

V. Conclusion and Future Works

In this study, based on the concept of sectorization, a new model is suggested for a LAP, in which the selection of DCs and the assignment of customers to them are done at the same stage. Its difference with a model

TABLE II
Obtained results for the benchmarks

Benchmark size	τ_{equ}	f
30 × 5	0.79	1.9e+03
450 × 75	0.98	5.92e+05
900 × 150	0.99	2.37e+06

from the literature that solves the same problem in two stages is discussed. The objective function of the model is defined based on the equilibrium of the distances in sectors. Different from the studies in the literature, the accessibility of customers from DCs is managed with a constraint, which is defined based on the covering radius instead of the compactness concept. The advantages of this definition are: (i) it is a simple linear constraint that can be included in solvers easily, uses a measure of accessibility for each customer, rather than the total distance within the sectors, and (iii) in real-life problems, it can be easily interpreted in terms of accessibility.

This study also has some limitations. For example, $\bar{D}$ in objective function 3 and $\bar{D}e$ in Constraint 11 are defined independently from the decision variables. Therefore, they are parameters and not variables. This is because the Pulp library could not find results for the case where they are declared as variables. It may be possible to find the result for this case with a different solver. In addition, although it is defined as $n_{max} \leq \bar{J}$ in Constraint 10, it is used as $n_{max} = \bar{J}$ in the experimental results. In future studies, more comprehensive results will be presented considering this matter.

In future studies, new models will be proposed for the multi-objective version of the problem.

Acknowledgements

This work is financed by the ERDF - European Regional Development Fund through the Operational Programme for Competitiveness and Internationalisation - COMPETE 2020 Programme and by National Funds of the Portuguese funding agency, FCT - Fundação para a Ciência e a Tecnologia within project POCI-01-0145-FEDER-031671.

References

[1] M. Tomintz, G. Clarke, and N. Alfadhli, "Location-allocation models. In Geocomputation," SAGE Publications, Inc., pp. 185-198, 2015.

[2] S. Barreto, C. Ferreira , J. Paixão, and B. S. Santos, "Using clustering analysis in a capacitated location-routing problem," European Journal of Operational Research, 179(3), pp. 968-977, 2007.

[3] M. Camacho-Collados, F. Liberatore, and J. M. Angulo, "A multi-criteria police districting problem for the efficient and effective design of patrol sector," European Journal of Operational Research, 246(2), pp. 674-684, 2015.

[4] O. V. Degtyarev, V. N. Minaenko, and M. O. Orekhov, "Solution of sectorization problems for an air traffic control area. Basic principles and questions of airspace sectorization and its formalization as an optimization problem," Journal of Computer and Systems Sciences International, 48(3), pp. 384-400, 2009.

[5] I. S. Litvinchev, G. Cedillo, and M. Velarde, "Integrating territory design and routing problems," Journal of Computer and Systems Sciences International, 56(6), pp. 969-974, 2017.

[6] A. Martinho, E. Alves, A.M. Rodrigues, J.S. Ferreira, "Multicriteria Location-Routing Problems with Sectorization," In: Vaz A., Almeida J., Oliveira J., Pinto A. (eds) Operational Research. APDIO 2017. Springer Proceedings in Mathematics & Statistics, vol 223. Springer, Cham, 2018.

[7] A.M. Rodrigues, J.S. Ferreira, "Measures in Sectorization Problems," In: Póvoa A., de Miranda J. (eds) Operations Research and Big Data. Studies in Big Data, vol 15. Springer, Cham, 2015.

[8] A. M. Rodrigues, and J. S. Ferreira, "Sectors and routes in solid waste collection," In Operational Research Springer, Cham, pp. 353-375, 2015.

[9] A. M. Rodrigues, and J. S. Ferreira, "Waste collection routing—limited multiple landfills and heterogeneous fleet," Networks, 65(2), pp. 155-165, 2015.

[10] K. Zhang, H. Yan, H. Zeng, K. Xin, and T. Tao, "A practical multi-objective optimization sectorization method for water distribution network," Science of The Total Environment, 656, pp. 1401-1412, 2019.

[11] A. Teymourifar, A. M. Rodrigues, and J. S. Ferreira, "A Comparison between NSGA-II and NSGA-III to Solve Multi-Objective Sectorization Problems based on Statistical Parameter Tuning," In 2020 24th International Conference on Circuits, Systems, Communications and Computers (CSCC), IEEE, pp. 64-74, 2020.

[12] A. Teymourifar, A. M. Rodrigues, and J. S. Ferreira, "A Comparison Between Simultaneous and Hierarchical Approaches to Solve a Multi-Objective Location-Routing Problem," Graphs and Combinatorial Optimization: from Theory to Applications, CTW2020 Proceedings, 251-263, 2021.

[13] V. Romanciuc, C. Lopes, A. Teymourifar, A. M. Rodrigues, and J. S. Ferreira, C. Oliveira, E. G. Öztürk, "An Integer Programming Approach to Sectorization with Compactness and Equilibrium Constraints," In: Machado J., Soares F., Trojanowska J., Ivanov V. (eds) Innovations in Industrial Engineering. icieng 2021. Lecture Notes in Mechanical Engineering, Springer Cham, pp 185-196, 2022.

[14] A. Teymourifar, A. M. Rodrigues, and J. S. Ferreira, C. Lopes, C. Oliveira, V. Romanciuc, "A Two-Stage Method to Solve Location-Routing Problems Based on Sectorization," In: Machado J., Soares F., Trojanowska J., Ivanov V. (eds) Innovations in Industrial Engineering. icieng 2021. Lecture Notes in Mechanical Engineering, Springer Cham, pp 148-159, 2022.

[15] M. Bashiri, and F. Fotuhi, "A cost-based set-covering location-allocation problem with unknown covering radius," In 2009 IEEE International Conference on Industrial Engineering and Engineering Management, IEEE, pp. 1979-1983, 2009.